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# The Appearance of a Rapidly Moving Object

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## ABSTRACT

We have investigated the observation, by means of radar, of a cube with edges of length  $l$  moving with a relativistic velocity ( $v$ ) relative to the observer. The special theory of relativity indicates that the cube should appear Lorentz contracted. If the observer takes a picture using a camera, the cube appears stationary and rotated through an angle  $\Delta\theta = \cos^{-1}(v/c)$  relative to the observer. In the radar picture the cube appears Lorentz contracted if the angular velocity,  $\omega$ , of the radar beam is much greater than  $v/r$ , where  $r$  is the distance of the cube from the observer.

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## INTRODUCTION

We shall consider a cube with edges of length 1 (measured in a frame at rest with respect to the cube) moving with a velocity  $v$  relative to an observer's frame of reference. Einstein's special theory of relativity leads one to think that one would see this cube contracted along the direction of its velocity. It has been shown <sup>1,2</sup> that a picture taken with a camera by the observer would not show a contraction. The cube would appear as if it were stationary and rotated through an angle  $\Delta\theta = \cos^{-1}(v/c)$  relative to the observer. The apparent rotation is due to light leaving different parts of the cube at different times in order to arrive at the camera at the same instant. We shall investigate the radar picture of the rapidly moving cube and see if the Lorentz contraction is observable.

## OBSERVER USING A CAMERA

A cube with sides AB, BC, CD and DA is moving with a velocity  $v$  relative to the frame of reference of an observer. The sides BC and DA (see figure 1) would be measured by the observer as  $l\gamma$  where  $\gamma = [1 - (v/c)^2]^{-\frac{1}{2}}$ . The sides AB and CD would be measured as 1. If we assume that the light from all parts of the cube arrive at the observer's camera at the

same instant, then the light from A, for example, must have left the cube before the light from B. The light from the point A would be emitted when the cube was a distance  $\Delta x = 1 \cdot (v/c) \cos \theta$  to the left of where the light is emitted from B (assuming that the distance to the observer is great compared to the size of the cube so that the light rays are essentially parallel.) The effect of the difference in the point of emitting the light and the Lorentz contraction combine to give a picture of the cube to the observer which appears to be stationary and rotated through an angle  $\Delta \theta = \cos^{-1}(v/c)$  relative to the observer. Thus, the Lorentz contraction is not observed.

#### OBSERVER USING RADAR

Let us again consider the cube with sides AB, BC, CD and DA. We shall observe the cube with a radar beam rotating counter-clockwise with an angular velocity  $\omega$  radians per second. Thus, a target with plane polar coordinates  $r$  and  $\theta$  will be detected when the beam is directed at an angle  $\theta$  and the distance,  $r$ , will be given by  $r = \frac{1}{2} c \tau$ , where  $\tau$  is the time between the emitted and received pulse as measured by the radar operator. In the case of the cube, the targets to be detected are the four edges (points in Figure 2) A, B, C, and D. We shall label the four points at which the edges

are detected as 1, 2, 4 and 3 respectively. If the time between the interception of the points A and B of the cube by the radar beam is  $\Delta t_1$ , then the co-ordinates of the interception of B are

$$r_2 = r_1 - l \cos \theta_1 - v \Delta t_1 \sin \theta_1$$

$$\text{and } \theta_2 = \theta_1 + \omega \Delta t_1 .$$

We have assumed the length of the cube is small compared to the distance to the observer.

The condition that the co-ordinates of the radar interception point of point B be the co-ordinates of B at the interception gives the relation

$$v \Delta t_1 \cos \theta_1 + \omega r_2 \Delta t_1 = l \sin \theta_1 .$$

The time  $\Delta t_1$  is therefore

$$\Delta t_1 = l \sin \theta_1 / (v \cos \theta_1 + \omega r_2) .$$

The co-ordinates  $r_2$  and  $\theta_2$  may then be expressed as

$$r_2 = r_1 - l \cos \theta_1 - v l \sin^2 \theta_1 / (v \cos \theta_1 + \omega r_2)$$

and

$$\theta_2 = \theta_1 + \omega l \sin \theta_1 / (v \cos \theta_1 + \omega r_2) .$$

The co-ordinates of the interception of D (point 3) by the radar beam will be

$$r_3 = r_1 + l \gamma \sin \theta_1 - v \Delta t_2 \sin \theta_1$$

and  $\theta_3 = \theta_1 + \omega \Delta t_2$ .

The time  $\Delta t_2$  is the time between interception of point A and point D and  $\gamma = [1 - (v/c)^2]^{-\frac{1}{2}}$ .

The Lorentz contraction along the direction of motion is used and the time  $\Delta t_2$  is

$$\Delta t_2 = l \gamma \cos \theta_1 / (v \cos \theta_1 + \omega r_3)$$

The co-ordinates  $r_3$  and  $\theta_3$  become

$$r_3 = r_1 + l \gamma \sin \theta_1 - v l \gamma \sin \theta_1 \cos \theta_1 / (v \cos \theta_1 + \omega r_3)$$

and

$$\theta_3 = \theta_1 + \omega l \gamma \cos \theta_1 / (v \cos \theta_1 + \omega r_3)$$

Similarly the co-ordinates of the interception of C (point 4) will be

$$r_4 = r_1 + l \gamma \sin \theta_1 - l \cos \theta_1 - v l \sin \theta_1 (\sin \theta_1 + \gamma \cos \theta_1) / (v \cos \theta_1 + \omega r_4)$$

and  $\theta_4 = \theta_1 + \omega l (\sin \theta_1 + \gamma \cos \theta_1) / (v \cos \theta_1 + \omega r_4)$ .

If we plot the four points in a  $r, \theta$  diagram, we will in general have a parallelogram with a base

$$b = l \gamma [1 - v \cos \theta_1 / (v \cos \theta_1 + \omega r_1)]$$

height  $h = l$  and angle between the sides

of  $\phi = \tan^{-1} [v \sin \theta / (v \cos \theta + \omega r)]$ . The cube will therefore be seen as a Lorentz contracted cube (rectangle) if  $\omega \gg (v/r)$ . If the radar operator determines the velocity of the cube, he may correct the shape of the object seen by using non-relativistic formulae. He would then come to the conclusion that he is looking at a rectangle with height  $l$  and length  $l\gamma$ . The contraction may then be seen even if  $\omega$  is not greater than  $v/r$ . The above formulae may be applied to points on a sphere and the sphere would then be seen as contracted along the direction of motion.

#### CONCLUSION

A cube with edges of length  $l$  moving with a velocity  $v$  relative to an observer will appear stationary, uncontracted and rotated through an angle  $\Delta\theta = \cos^{-1} (v/c)$  with respect to an observer using a camera. The cube will appear Lorentz contracted along the direction of motion and unrotated if observed using radar with the angular velocity,  $\omega$ , of the radar beam much greater than  $v/r$ . The contraction will also appear undistorted if the radar operator determines the relative velocity of the cube and corrects for the non-relativistic distortion. If one carries out the above analysis for a spherical object, the sphere will also be seen contracted along the direction of motion.

- 1, J. Terrell, Phys. Rev., 116, 1041 (1959)
- 2, V. F. Weisskopf, Physics Today, 13, No. 9, p.24 (Sept., 1960)



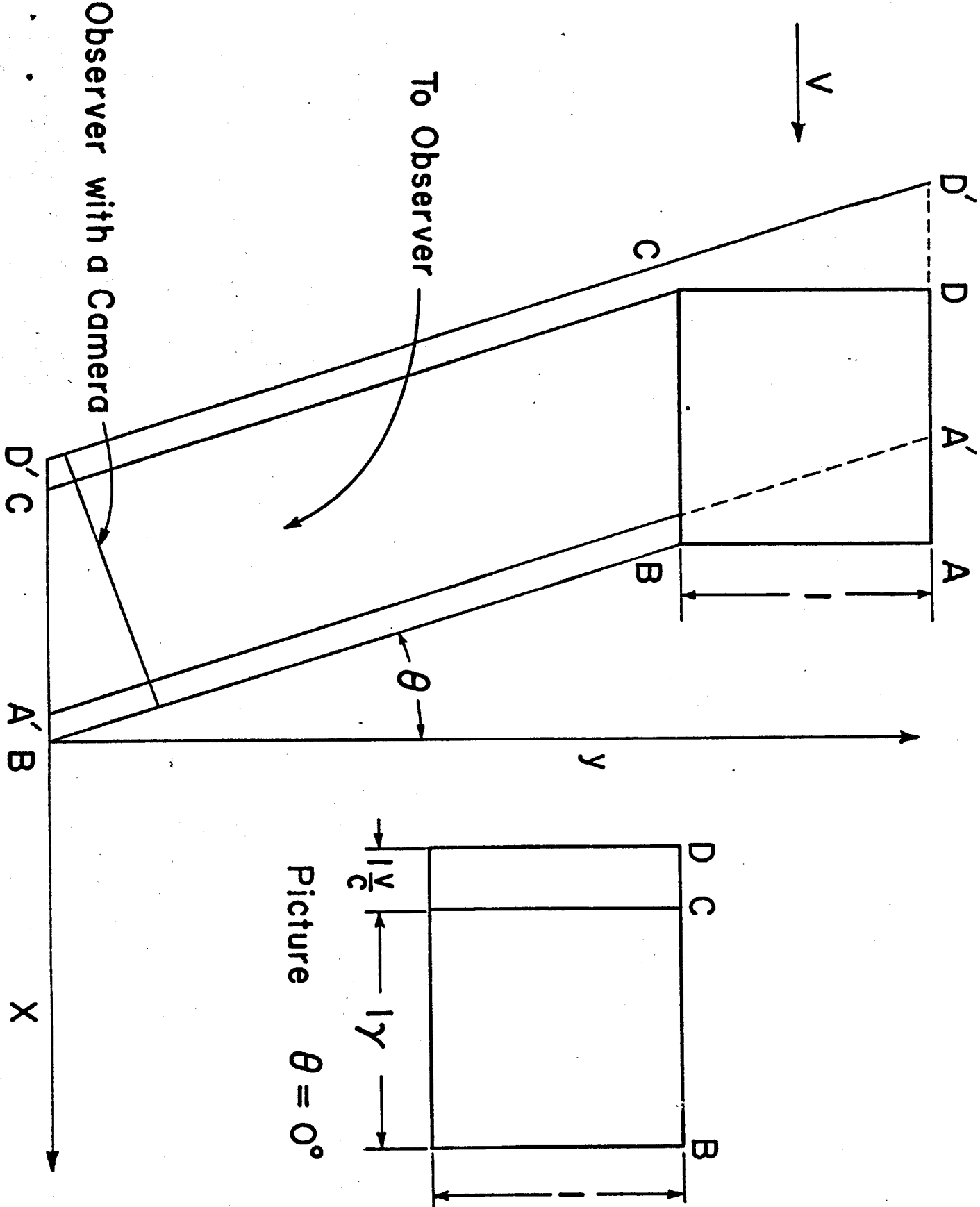
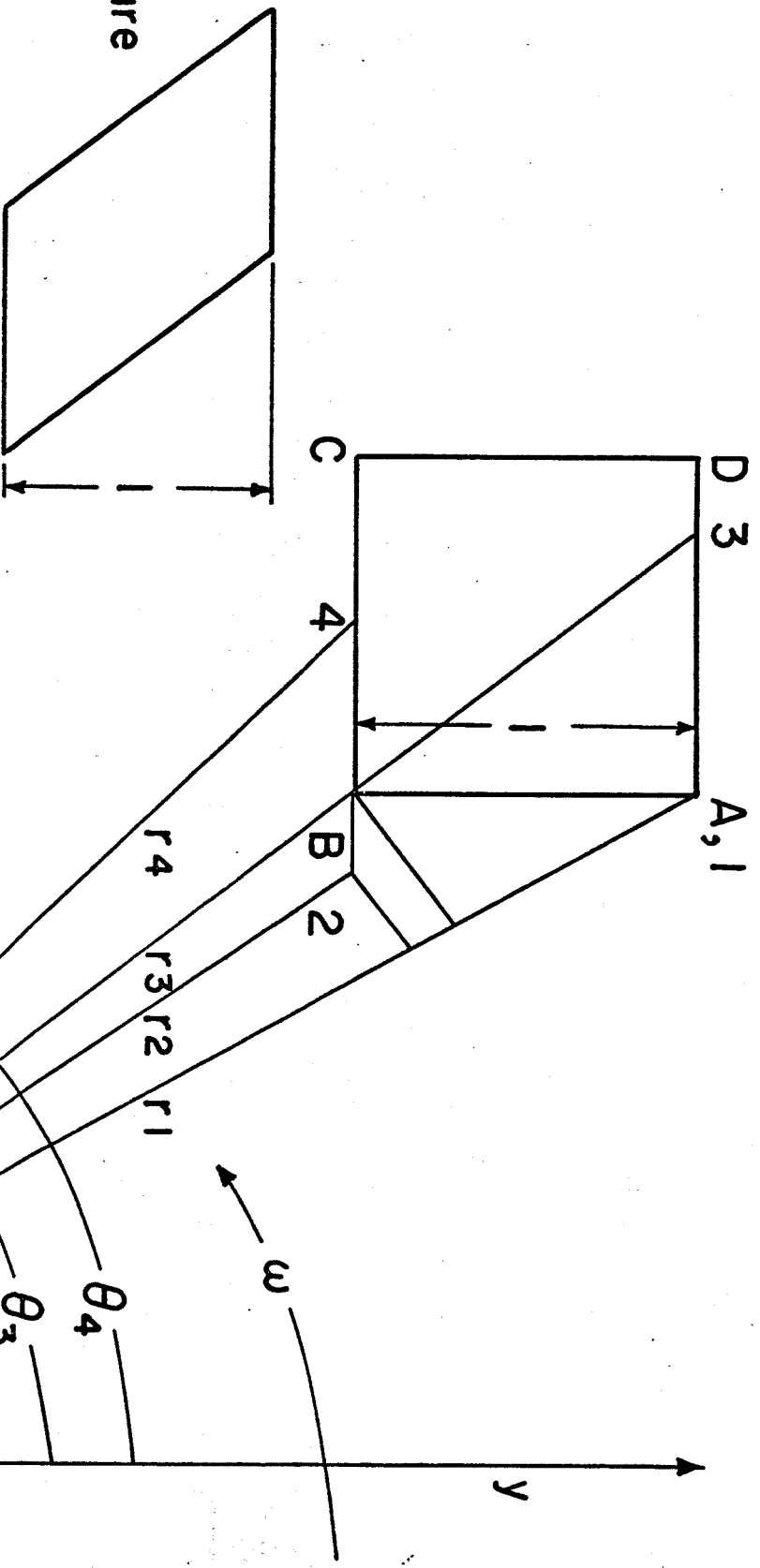
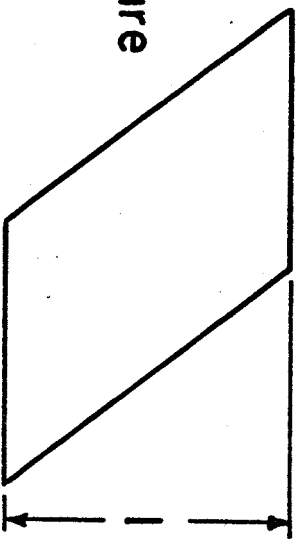


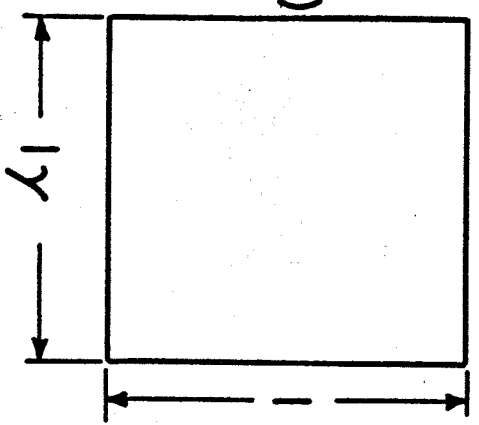
Fig 1.



General Picture



Picture if  $\omega \gg (v/r_1)$



Observer with Radar

